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THE DESIGN OF SUPERMANEUVERABLE FIGHTER AIRCRAFT, HIGH-PRECISION SPACE-BORN OPTICAL TRACKING SYSTEMS AND TRANSATMOSPHERIC HYPERVELOCITY VEHICLES WILL POSE SIGNIFICANT CHALLENGES TO MODERN CONTROL SYSTEM DESIGN THEORY. THE THEME OF THE RESEARCH HAS BEEN "MAKING MODERN CONTROL THEORY WORK." THE PRODUCT OF THE RESEARCH HAS BEEN THEORY, ALGORITHMS AND SOFTWARE APPLICABLE TO MULTIVARIABLE FEEDBACK CONTROL PROBLEMS IN WHICH THERE ARE DESIGN CONSTRAINTS REQUIRING ROBUST ATTAINMENT OF STABILITY AND CONTROL PERFORMANCE OBJECTIVES IN THE FACE OF BOTH STRUCTURED AND UNSTRUCTURED UNCERTAINTY.

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FINAL REPORT
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PRACTICAL METHODS FOR ROBUST MULTIVARIABLE CONTROL

August 1, 1988 - July 31, 1989

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ABSTRACT

The design of supermaneuverable aircraft, high-precision space-born optical tracking systems and other aerospace systems poses serious challenges to modern control system design theory. The theme of the proposed research is "making modern control theory work." The product of the research has been theory and algorithms that can be applied to practical feedback design problems in which there are specifications requiring robust performance, insensitivity and stability in the face of both structured and unstructured uncertainty. New theory and methods have been developed to enable one to deal effectively and systematically with more realistic performance specifications than was previously possible. Included among these advances have been improved algorithms for computing reduced-order models for use in control system design, methods for exactly determining the robustness of stability in the face of several uncertain real parameters, a less conservative method for testing the stability of systems with a single sloped-bounded nonlinearity, and several key theoretical and algorithmic advances relating to robust H^∞ optimal control system design. Current research (being supported under AFOSR Grant 89-0398) is continuing in this vein, focusing on numerical robustness and algorithmic efficiency issues in H^∞ robust control synthesis and relative-error model (REM) reduction, developing practical methods for "real k_m" multivariable stability margin analysis,

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expanding the class of systems which the H^∞ robust control theory can handle, and developing singular-value oriented system identification algorithms suitable for use in robust control system design.

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1.* E. Jonckheere and R. Li, "Hankel Operator and H_∞ Distance Problem Over a Simply-Connected Domain," to appear Int. J. Control, 1990.

2.* J. Bar-on and E. A. Jonckheere, "Phase Margins for Multivariable Control Systems," submitted to Int. J. Control, 1989.

3.* M. G. Safonov and H. Flashner, "Modeling and Robustness Issues in Control Design for Flexible Structures," presented at 1989 American Control Conference, Pittsburgh, PA, June 21-23, 1989.

4.* G. P. Papavassilopoulos and M. G. Safonov, "Robust Control Design Via Game Theoretic Methods," Proc. IEEE Conf. on Decision and Control, Tampa, FL, December 13-15, 1989.

5.* B. R. Copeland and M. G. Safonov, "Two-Riccati H^∞ Controllers for Descriptor Systems," Proc. American Control Conf., Pittsburgh, PA, June 21-23, 1989.

6.* M. G. Safonov, R. Y. Chiang and H. Flashner, "H^∞ Robust Control Synthesis for a Large Space Structure," submitted to AIAA Journal on Guidance, Control and Dynamics, June 1989.

7.* M. G. Safonov and R. Y. Chiang, "A Schur Method for Balanced-Truncation Model Reduction," IEEE Trans. on Autom. Control, Vol. 34, No. 7, July 1989.

8.* V. X. Le and M. G. Safonov, "Rational Matrix GCD's and the Design of Squaring-Down Compensators - A State-Space Theory," submitted to IEEE Trans. on Autom. Control, July 1989.

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INTRODUCTION: THE PROBLEM

The underlying problem in robust feedback control system synthesis is to find a feedback controller C(s) such that a given vector, say col (e,u,y), whose components comprise the control system's error, control and plant output signals, respectively, remains in a specified set despite uncertain disturbances, parameters, gains, phases and nonlinearities within a given set, say $\underline{D}$. The performance specifications on the signals e, u, and y may be expressed in terms of frequency response inequalities (for broadband r.m.s. disturbance rejection), closed-loop pole locations (for acceptable transient response to impulsive and step disturbances), closed-loop zero locations (for asymptotic tracking and asymptotic rejection of disturbances with known poles).

It turns out that this general problem can be reformulated as a consequence a certain lemma of Youla as the problem of finding the set, say $\underline{X}$, of all transfer function matrices X(s) having "stable" poles (i.e., poles in a stipulated region) for which the <u>excess stability margin</u> k_m satisfies

$$k_m(A + BXC; \underline{D}) > 1 \tag{1}$$

(see [22,23] and the references therein). Here the A(s), B(s), and C(s) are transfer function matrices which depend on the specific plant and on where the uncertain noises, parameters, etc., enter. The <u>multivariable stability margin</u> function $k_m(T;\underline{D})$ is defined for any transfer function matrix T(s) and any set $\underline{D}$ of uncertain operators as [40]

$$k_m(T;\underline{D}) = \inf \{k:\ k \text{ real, } (I + kDT)^{-1} \text{ is "unstable" for some } D \text{ in the set } \underline{D}\}\ ; \tag{2}$$

the quantity $1/k_m$ has been called the <u>structured singular value</u> $\mu(T)$ by Doyle [26]. Thus, $k_m(T;\underline{D})$ is the gain margin (for the worst-case D in the set $\underline{D}$) of a hypothetical feedback system having loop transfer function T. The quantity $k_m(T,\underline{D})$ is defined to be zero when T is open loop unstable. The notion of "unstable" is left intentionally vague here, since the appropriate definition of stability may vary depending on the application. For example, it may refer to stability with a specified degree, e.g., with all poles in some specified set [58]. A "stable" function X(s) (that is, a stabilizing compensator C(s)) verifying (1) achieves the ultimate design objective, but one may also look at optimizing the performance as

$$k_m^{opt} := \max_{X \text{ "stable"}} k_m(A + BXC;\underline{D}) . \qquad (3)$$

Currently, it is practical to compute the function $k_m(. \, ; \, .)$ only in special cases such as when the set $\underline{D}$ is finite or when $\underline{D}$ is the set of all transfer function matrices whose largest singular value is bounded for all frequency by a given number, i.e., when $\|D\|_\infty$ is bounded, in which case the problem (3) reduces to the multivariable L^∞ optimization problem [22,23]

$$k_m^{opt} := \min_{X \text{ "stable"}} \| A + BXC \|_\infty . \qquad (4)$$

The problem of developing a useful characterization of the set $\underline{X}$ of transfer function matrices X(s) satisfying (1) likewise can only be solved in special cases, e.g., $\underline{D}$ singular-value bounded or $\underline{D}$ real, scalar gains. Also unsolved, and not less difficult, is the problem of optimizing the k_m-performance as described by (3). Our research over the past few years has addressed these unsolved problems, building upon and extending the theoretical base of H^∞ optimal control theory. We have made significant strides toward our goal of creating a cohesive body of theory that may be used by engineers to solve the broadest possible class of practical robust multivariable feedback control design problems.

SUMMARY OF PROGRESS PREVIOUSLY REPORTED

Since our AFOSR-supported research under Grants 85-0256 and 88-0282 began in July 1985, progress has been made on several aspects of this problem, leading to a substantial number of AFOSR-supported reports and publications [1-20, 24, 28-31, 34, 56-61]. Among the new results is a vastly improved "Toeplitz + Hankel" algorithm for computing the minimal cost for L^∞ optimal control problems [3,5,14,16,17]; the results promise to reduce computer-time for L^∞ control calculatios by a factor of 10. Another result [18] involves a vector-valued alternative to the standard L^∞ control problem which has been bound to enable a more precise trade-off between sensitivity S(s) and complementary sensitivity I-S(s). In [4,5] we describe how the frequency-weighted LQG (Linear Quadratic Gaussian) synthesis theory (Safonov et al. [25]) was used to design a robust multivariable controller for a 40-state model of a flexible mechanical truss structure; the control design worked well when digitally implemented and connected to the infinite-order real system. In [2] a homotopy method for eliminating conservativeness in $\mu(T;D)$ stability margin calculation was developed and evaluated, but found to be too computationally demanding to be practical. Further study resulted in a significant breakthrough in nonconservative $\mu(T;D)$ calculation techniques in [1,7,19]; these new results make computation of $\mu(T;D)$ practical for the first time for the important case when the set $\underline{D}$ is a cube in $\mathbb{R}^n$ (i.e., the case of several uncorrelated unknown-but-bounded uncertain real parameters); this problem has become popularly known as the "real k_m" or "real μ" problem. A major practical advance in 1986 was the development at USC of a software package [8] within the CTRLC^{TM}/PC-MATLAB^{TM} framework for solving a broad class of L^∞ optimal control problems. Over the past year, in further work not supported by AFOSR, we have collaborated with the publishers of PC-MATLAB to create a new <u>PC-MATLAB Robust-Control</u>

Toolbox, software package and user's guide [65]. Our toolbox makes the L^∞ optimal control theory and associated Hankel and balanced model reduction theory widely accessible to practicing engineers.

The process of developing and testing this software enabled us to identify and resolve a number of minor, but critical, shortcomings of the existent L^∞ conceptual algorithms; the initial versions of the refined L^∞ theory and algorithms were summarized in [15]. An early version of our Robust-Control Toolbox called LINF was used for a "benchmark" multivariable aircraft controller design problem in [9] and for a flexible space structure controller design in [65]. In a separate development, we developed a significantly improved computer-oriented criterion for nonlinear stability which may render the celebrated Popov criterion obsolete; our new nonlinear stability criterion is superior (i.e., less conservative than) the standard graphical criteria including the circle criterion, the off-axis circle criterion, and the Popov criterion. Another major breakthrough has been the solution of the diagonally-scaled H^∞ optimal control problem for a limited but nontrivial class of problems [10,12,30]; this new theory enables achievement of our ultimate design objective, namely the solution of (3) for a limited class of problems involving complex structured uncertainty.

PROGRESS THIS YEAR

In the past year, we have made several major advances in the area of H^∞ optimal control theory, in algorithms for model order reduction and in the mathematical system theory. We regard the first two of the following to be major practical advances, and the third has been a major theoretical advance; the latter three are important contributions to the theoretical infrastructure but they are still too new to accurately gauge their practical impact:

1. Two-Riccati H^∞ Formulae [36, 56, 60]

2. Basis-Free Model-Reduction Formulae [24, 28, 57]

3. Spectral Theory of LQ and H^∞ Problems [59-61]

4. Phase Margin for Multivariable Systems

5. H^∞ Control Over Arbitrary Regions of the Complex Plane [58]

6. State space formula for matrix GCD's.

Two-Riccati H^∞ controller formula, developed in a series of papers by Doyle, Glover, Limebeer, Kasenally and Safonov [56,62,38,39,63,64] and closely related to the formula of Juang and Jonckheere [35, 36], constitute what may be the single greatest breakthrough in control theory in the past decade. These formula enable one to completely bypass the Youla parameterization A + BXC and solve the multivariable H^∞ optimization problem (4) by solving two Riccati equations of the state-space (A, B, C, D) matrices of the plant. The result is the two-Riccati formula for "order n" H^∞ controllers which are no more complicated to compute or implement than H^2 controllers (i.e., LQG controllers). We have coded these formula using PC-MATLAB and CTRL-C and found them to be superior for computer implementation of H^∞ optimal control theory, producing H^∞ controller solutions reliably for plants with dozens of states in only a few minutes of computer time on a VAX 11/780 and on a SUN 3/50 workstation.

We have pursued the "two-Riccati" breakthrough in the H^∞ theory further in [60,61]. In [60], we develop an embedding technique involving "loop shifting" variable changes which enable the general H^∞ optimal control problem to be reduced to the much simpler special case intially treated by Doyle et al. [38, 39, 63]. The simplications made possible by our loop shifting techniques made it practical, for the first time, to present complete derivations of the H^∞ theory for the general case. In computer studies we have also observed that the loop-shifting formula are easier to code and slightly faster to compute with than the equivalent two-Riccati general formulae of Glover et al. [64] and Limebeer et al. [56].

The second major advance, our basis free model reduction formulae [24, 28, 57], has made model order reduction with an infinity-norm error-criterion practical for those systems which stand to benefit the most from model reduction, viz., systems with some modes which are nearly uncontrollable or nearly unobservable. Though perhaps not particularly exciting from a purely theoretical point of view, they are a major advance because they make Hankel Optimal (HO) model reduction, Balanced Truncation (BT) model reduction and Balanced Stochastic Truncation (BST) model reduction practical. A critical shortcoming of these three methods that had gone unnoticed by theoreticians heretofore precluded their use on systems with uncontrollable or unobservable modes. A previously required "first step" in these infinity-norm criterion model reduction methods involved finding a "balancing transformation," a transformation which generically fails to exist for non-minimal realizations. Theoreticians failed to recognize the problem since, in theory, one can always eliminate non-minimal modes. In practice, however, systems are generically observable and controllable, even if only barely so, and, in practice, one of the primary uses of model reduction is to identify and discard the barely observable/controllable modes. Moreover, a computer with finite numerical precision cannot distinguish a barely observable mode from an unobservable one and, in any case, some "barely observable" modes can turn out to have a very significant impact on the frequency-response of a system. Thus, it is folly to suppose, as the previous literature had, that one can usefully begin a model reduction procedure by discarding the unobservable and uncontrollable modes. Our basis-free methods for model reduction bypass the inherently ill-conditioned initial balancing step. The resulting model reduction formula are simpler, faster to compute, and most importantly they work. They work even for nonminimal and nearly nonminimal systems, reliably eliminating the unobservable and uncontrollable modes while ensuring that the important infinity-norm error bounds associated with Hankel, balanced truncation and balanced stochastic truncation model reduction methods are satisfied.

The relative-error infinity-norm error bounds of BST makes our basis free BST algorithm in [57] especially attractive for robust control system design. A "robustness theorem" [57] establishes that a model is useful for designing feedback control systems only if its relative error is less than one thoughout the control loop bandwidth as determined from singular-value Bode plots of the loop transfer function matrix. This robustness theorem proved vital in our TRW-supported large space-structure design study [65] in which a 4-state plant model surprisingly was proved to be adequate for a structure having 116 modes within the control loop bandwidth. This work is a spinoff of the so-called "phase matching" problem initiated by Jonckheere; see, e.g., [48], [49] and references therein.

The "Toeplitz + Hankel" operator theoretic interpretation of the H^∞ theory has led to a number of theoretical insights into the H^∞ optimal control problem which we hope will eventually lead us to better and faster computational algorithms and, perhaps, to generalization of the H^∞ control theory. Moving beyond our early work on fast Toeplitz + Hankel algorithms [3,14,16,17,34,35], our recent work in [59,61] achieves, we feel, a complete understanding of the links between the H^∞ problem and the spectral theory of the linear-quadratic problem. In a few words, this is the essence of the results in [59,61]:

Consider the standard 2-block frequency response inequality

$$\left\| \begin{matrix} H(j\omega) - Q(j\omega) \\ V(j\omega) \end{matrix} \right\| \leq \varepsilon, \ \forall\, \omega$$

verified for some

$$Q \in H^{\infty}{}_{\perp}$$

where

$$\begin{pmatrix} H(s) \\ V(s) \end{pmatrix} = \begin{pmatrix} D_H \\ D_V \end{pmatrix} + \begin{pmatrix} C_H \\ C_V \end{pmatrix} (sI - A)^{-1}B \in H^{\infty}$$

The key idea is to map the frequency response inequality to the time domain using Parseval's like arguments. This yields

$$\int_{-\infty}^{0} (x^T \; u^T) \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} \begin{pmatrix} x \\ u \end{pmatrix} \leqslant \varepsilon^2 \int_{-\infty}^{0} u^T u \; dt, \; \forall \; u$$

where x is generated by the state space equation

$$\dot{x} = Ax + Bu$$

and

$$Q = -C^T{}_H C_H$$

$$R = D^T{}_V D_V$$

$$S = (Y_H + Y_V)B_H + C^T{}_V D_V$$

where

$$A^T(Y_H + Y_V) + (Y_H + Y_V)A = -(C^T{}_H C_H + C^T{}_V C_V)$$

The cornerstone of the spectral theory of the linear quadratic problem -- proved ten years ago by Jonckheere and Silverman -- is that

$$\int_{-\infty}^{0} (x^T \ u^T) \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} \begin{pmatrix} x \\ u \end{pmatrix} = (u_1, (T + H^*{}_1 H_2) u)$$

where T is Toeplitz and H is Hankel.

Using the LQ-H^∞ mapping, all the results of the spectral theory of the linear quadratic problem have an H^∞ interpretation, and vice-versa. Consequently, this symbiotic LQ/H^∞ theory has allowed to provide simple linear-qudratic insight to such problems as (i) degree of H^∞ compensator; (ii) pole/zero cancellation at H^∞ optimality; (iii) Riccati equation solution to H^∞ design; (iv) γ-iteration, etc. The challenge before us, now that we understand these relationships, will be to turn these operator-theoretic insights into practical algorithms. This is one of the aims of our current work.

The most significant practical impact of this symbiotic LQ/H^∞ theory, which we were first to introduce [3], is a better understanding of the termination condition on the γ-iteration. Indeed, in the 2-Riccati solution to the 4-block problem, the tolerance level γ is recursively decreased until "something" breaks down in the Riccati construction of the compensator achieving the tolerance γ. With the LQ/H^∞ theory at hand, we relate the breakdown of the 2-Riccati equation construction of the compensator to the spectral structure of several "Toeplitz + Hankel" operators. Depending on whether γ hits the continuous or discrete spectrum, the Riccati solution either has closed loop poles on the $j\omega$-axis or has the wrong sign. Finally, if optimality is achieved at the discrete spectrum, an easy procedure for reducing the size of the H^∞ compensator emerges.

The fourth area in which we have made progress in the past year is in the development of a more appropriate concept of <u>multivariable phase margin</u> [90]. The notion of phase margin is of paramount importance since it measures the amount of phase lag a loop can tolerate before going unstable. The notion of phase margin is well understood and universally accepted for Single-Input-Single-Output systems. However, despite many efforts, there has not until now been a completely satisfactory definition of phase margin for multivariable systems.

In the course of this research, we have developed a new notion of phase margin for multivariable systems. The driving motivation is spacecraft attitude control. Typically, if the sensor axes are not properly aligned with either the body axes or the inertial axes, this will introduce an uncertain (unitary) rotation transformation in the feedback channel. This is in most cases also combined with delays in the sensors. The combination of sensor delay and axis misalignment results in a unitary uncertainty in the feedback channel.

It is therefore necessary to define a measure of robustness against unitary perturbation in the feedback channel. This measure of robustness is precisely our notion of multivariable phase margin. Roughtly, the multivariable phase margin is the maximum allowable phase of a unitary perturbation Δ that remains stabilizing.

More precisely, by definition, the phase of the unitary perturbation Δ is the largest absolute value of the arguments of its eigenvalues, the arguments being resolved within $(-\pi, \pi]$.

The phase margin of the loop matrix L, $\Phi(L)$, has the remarkable property that

$$\forall \text{ unitary } \Delta$$

such that

$$\text{phase}(\Delta) < \Phi(L)$$

the loop $L\Delta$ is stable.

Furthermore, there is a weak converse to it. Namely, for any phase angle ϕ greater than

$\Phi(L)$, there exists a destabilizing unitary perturbation Δ with phase equal to ϕ.

An algorithm to compute $\Phi(L)$ has been developed and implemented on a PC. Interestingly, there are several ways to look at the computational implementation of the phase margin problem. One way is to look at it as a programming problem (a quadratic optimization with quadratic constraints). Another way is to look at it as a two variable spectral problem, from which it follows that $\Phi(L)$ is equivalent to an algebraic curve problem.

In the course of computer simulation it became evident that the multivariable phase margin problem has a rich topology--namely, the issue appears to be the topology of the set of stabilizing Δ's in the manifold of all unitary perturbations, and whether these "stability islands" can be characterized by some metric.

In the case of two channels in the feedback path, and a special unitary perturbation, i.e., $\Delta \in SU(2)$, the phase margin takes the geometric interpretation of a Riemann metric on a manifold. The group SU(2) of special unitary matrices is isomorphic to the (multiplicative) group H of unit quaternions. The group of quaternions can be endowed with the so-called left-invariant metric that makes it a topological group. It turns out that the phase of the perturbation Δ is the left-invariant metric between the identity (under multiplication) quaternion $q = (1,0,0,0)$ and the quaternion representing Δ. Furthermore, the group of unit quaternions is itself isomorphic to the 3-sphere S^3 imbedded in the Euclidean space R^4. The left invariant metric between two quaternions represented as points on S^3 is nothing other than the arc of great circle (or geodesic line) joining the two points on the sphere.

Therefore, the relevant manifold of uncertainties in the case $\Delta \in SU(2)$ is the sphere S^3. The identity perturbation in the feedback channel corresponds to the "North Pole" of the sphere.

The set of stabilizing perturbations is a "polar cap" on S^3. Finally, the phase margin is the lowest "latitute" above which any perturbation is guaranteed to be stabilizing.

We feel this notion of the phase margin as a metric on a manifold of uncertainties could be extended to other robustness problems, i.e., the Doyle μ function.

Finally, we briefly discuss the fifth area in which we have made significant progress this past year: Model reduction and H^∞ control over a planar domain [58]. This work, which builds upon our earlier work in [21,67] provides state-space formula for solving "one-block" H^∞ optimization problems over a subset Ω of the complex plane specifiable in the form

$$\Omega = \{z \in \mathbb{C} \mid \sum_{ij} \gamma_{ij} \bar{z}^i z^j \geq 0\}.$$

In the solution to the $H^\infty(\Omega)$ problem, we have followed the conventional line of arguments. The first step is the rational coprime factorization, which is a purely algebraic object and carries over to any analyticity domain Ω. Then via some Ω inner-outer and Ω spectral factorizations, the problem is reduced to the so-called $H^\infty(\Omega)$ distance problem,

$$\min_{X \in H^\infty(\Omega)} \| H-X \|_{H^\infty(\Omega)}$$

$$(H(s) = C(sI - A)^{-1}B)$$

The above problem is solved via the so-called Mazko Lyapunov equations for the domain Ω, i.e.,

$$\sum_{i,j} \gamma_{ij} A^i P A^{jT} = Q$$

It is easily seen on a little thought that the Bartels-Stewart algorithm can be carried over to this generalized Lyapunov equation.

Finally, the compensator is obtained via classical back substitution.

It is important to observe that our approach does not rely on the Riemann mapping of Ω to the Left-Half-Plane (or the unit disk). This indeed would result in an inflation of dimensionality. Observe in the above that we have managed to keep the dimension down. Although Ω might be a complicated domain requiring a transcendental Riemann mapping function, the size of A appearing in the Mazko Lyapunov equation is simply the same as the degree of X in the $H^{\infty}(\Omega)$ distance problem.

The sixth and last area in which we have made significant progress is in the computation of matrix GCD's (greatest common divisors) [82,87]. It turns out that GCD computation is equivalent to the design of "squaring-down" compensators for multiloop feedback control. A squaring-down compensator is a non-square compensator transfer function matrix inserted in series with a non-square plant transfer function matrix so that their product is square. The key property that a good squaring-down compensator must possess is that it must not introduce any non-minimum-phase zeros. In [82] and [87] establish the equivalence between squaring down and GCD computation and develop state-space algorithms for performing GCD computations. We also show that squaring-down compensators facilitate the solution of certain "singular" H^{∞} control problems.

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